



**GCE**

**Further Mathematics A**

Unit **Y535**: Additional Pure Mathematics

Advanced Subsidiary GCE

**Mark Scheme for June 2018**

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2018

Y535

Mark Scheme

June 2018

## Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

**Subject-specific Marking Instructions for AS Level Further Mathematics A**

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.  
If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is

ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep\*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are

Y535

## Mark Scheme

June 2018

established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Y535

Mark Scheme

June 2018

Question		Answer	Marks	AO	Guidance
1	(i)	$\mathbf{b} - \mathbf{a} = \begin{pmatrix} 7 \\ 0 \\ 1 \end{pmatrix}, \mathbf{c} - \mathbf{a} = \begin{pmatrix} 10 \\ 4 \\ -1 \end{pmatrix}, \mathbf{c} - \mathbf{b} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ $(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a}) = \begin{pmatrix} -4 \\ 17 \\ 28 \end{pmatrix}$ $\text{Area } \triangle ABC = \frac{1}{2}   \text{their } (\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})  $ $= \frac{33}{2}$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>1.1a</b></p> <p><b>1.1a</b></p> <p><b>1.1</b></p> <p><b>1.2</b></p> <p><b>1.1</b></p>	<p>Any two of these</p> <p>Attempt at a suitable vector product</p> <p><b>FT</b> correct</p> <p><b>FT</b> in surd, rational or decimal form</p>
			[5]		
	(ii)	$AB = \sqrt{50} \text{ or } 5\sqrt{2}$ $\text{Area } \triangle ABC = \frac{1}{2} (AB) d = \frac{33}{2}$ $\Rightarrow d = \frac{33}{5\sqrt{2}}, \frac{33}{\sqrt{50}}, \frac{33\sqrt{50}}{50}, \frac{33\sqrt{2}}{10}$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>1.1</b></p> <p><b>1.1a</b></p> <p><b>1.1</b></p>	<p><b>FT</b> their <math>(\mathbf{b} - \mathbf{a})</math> from (i)</p> <p>Equating <math>\frac{1}{2} \times \text{base} \times \text{ht.}</math> (correct setting) to (i)'s answer</p> <p><b>cao</b> any exact surd form</p>
		[3]			

$3.3\sqrt{2}$  or  $\sqrt{21.78}$  also ok

Question		Answer	Marks	AO	Guidance	
2	(i)	$\frac{\partial z}{\partial x} = 18x^2 + 2xy$	M1 A1	2.1 1.1	Good attempt at 1 <sup>st</sup> partial derivative 1 <sup>st</sup> correct	Condone lack of visible check that $z = 0$ also
		$\frac{\partial z}{\partial y} = \frac{2}{9}y + x^2$	A1	1.1	2 <sup>nd</sup> correct	
	When $x = y = 0$ , both $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ are zero	B1	2.2a	Verified, checked or noted. FT provided both p.d.s have both $x$ and $y$ terms		
			[4]			
	(ii)	For $x, y \neq 0$ , $\frac{\partial z}{\partial x} = 0 \Rightarrow y = -9x$	M1 *	1.1a	Setting both first p.d.s equal to 0	
and $\frac{\partial z}{\partial y} = 0 \Rightarrow y = -\frac{9}{2}x^2$		A1	2.2a	FT Both preliminary statements correct provided both p.d.s have both $x$ and $y$ terms		
$-9x = -\frac{9}{2}x^2 \Rightarrow (x \neq 0) x = 2$  $(x, y, z) = (2, -18, 12)$		M1 dep. A1 A1	2.1 1.1 1.1	Eliminating and solving for $x$ FT $x$ correct  <b>cao</b>		
			[5]			
3		$a(4n + 1) - b(6n + 1)$ attempted $a = 3, b = 2$ so that $n$ is eliminated, "1" obtained so if $h = \text{hcf}(4n + 1, 6n + 1)$ then $h \mid 1 \Rightarrow h = 1$	M1 A1 A1	1.2 1.1 2.2a	A linear combination of the 2 terms  Given Answer properly deduced	$a, b$ must be integers
			[3]			
		Alt. Let $h = \text{hcf}(4n + 1, 6n + 1)$ . Then $h \mid \{6(4n + 1) - 4(6n + 1)\} = 2 \Rightarrow h = 1$ or 2 But $(4n + 1), (6n + 1)$ are odd, so $h \neq 2 \Rightarrow h = 1$	M1 A1 E1		Two parts required for the M	
	Alt. Euclidean Algorithm: $6n + 1 = 1 \times (4n + 1) + 2n$ $4n + 1 = 2 \times (2n) + 1$ $2n = 2n \times (1) + 0$ $h = \text{final non-zero remainder} = 1$	M1 A1 A1		Correct to at least here  Given Answer properly deduced		



Y535

Mark Scheme

June 2018

Question		Answer	Marks	AO	Guidance	
4	(i)	$A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $B^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	<b>B1</b> <b>B1</b>	<b>1.1a</b> <b>1.1</b>	<b>BC</b> <b>BC</b>	
			[2]			
	(ii)	Clear statement that $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is an element  $AB = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$ $BA = \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix} = (AB)^2$  $ABA$ or $BAB = \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$	<b>B1</b>  <b>B1</b> <b>B1</b>  <b>B1</b>	<b>2.2a</b>  <b>1.1a</b> <b>1.1</b>  <b>1.1</b>	Anywhere (e.g. in a Cayley table)  <b>BC</b>  <b>BC</b>	
			[4]			
	(iii)	<b>A</b> and <b>B</b> have order 2 <b>AB</b> and <b>BA</b> have order 3 <b>ABA</b> (or <b>BAB</b> ) has order 2	<b>B1</b> <b>B1</b> <b>B1</b>	<b>2.1</b> <b>1.1</b> <b>1.1</b>	Stated or shown	
			[3]			
(iv)	<i>G</i> not abelian since (e.g.) $AB \neq BA$  <i>G</i> not cyclic since no element of order 6 ( <b>or</b> not abelian $\Rightarrow$ not cyclic)	<b>E1</b>  <b>E1</b>	<b>2.4</b>  <b>2.4</b>	Must have been shown  Valid correct statement supported by their elements and orders	(not just claimed as a general result for matrix multn.)	
		[2]				

Y535

Mark Scheme

June 2018

Question		Answer	Marks	AO	Guidance
5	(i)	$M + 2N = 102a - 17b = 17(6a - b)$ If $17 \mid N$ then $17 \mid M = 17(\dots) - 2N$ If $17 \mid M$ then $17 \mid 2N = 17(\dots) - M \Rightarrow 17 \mid N$ since $\text{hcf}(2, 17) = 1$	<b>B1</b> <b>M1</b> <b>A1</b> <b>A1</b>	<b>1.1</b> <b>3.1a</b> <b>2.4</b> <b>2.1</b>	The factor of 17 must be noted (here or later) For valid $\Rightarrow$ argument For valid $\Leftarrow$ argument Condone lack of use of <i>Euclid's Lemma</i> to justify result
			[4]		
	(ii)	$2\ 058\ 376\ 813\ \underline{901} \rightarrow 20\ 583\ 768\ 139 - 9$ $\qquad\qquad\qquad = 20\ 583\ 768\ 130$ $20\ 583\ 768\ \underline{130} \rightarrow 205\ 837\ 681 - 270$ $\qquad\qquad\qquad = 205\ 837\ 411$ $205\ 837\ \underline{411} \rightarrow 2\ 058\ 374 - 99 = 2\ 058\ 275$ $2\ 058\ \underline{275} \rightarrow 20\ 582 - 675 = 19\ 907$  $19\ \underline{907} \rightarrow 199 - 63 = 136$ $136 = 8 \times 17$ and the original number $M$ is a multiple of 17	<b>M1 A1</b>  <b>M1</b>  <b>A1</b>	<b>1.1a</b> <b>1.1</b>  <b>1.1</b>  <b>1.1</b>	First step clearly and correctly demonstrated  Process repeated iteratively to a suitable stopping-point  All correctly demonstrated Final statement of result not required
			[4]		

Question		Answer	Marks	AO	Guidance
6	(i)	$F_{n+5} = F_{n+4} + F_{n+3} = (F_{n+3} + F_{n+2}) + F_{n+3}$ $= 2F_{n+3} + F_{n+2}$ $= 2(F_{n+2} + F_{n+1}) + F_{n+2}$ $= 3F_{n+2} + 2F_{n+1}$ $= 3(F_{n+1} + F_n) + 2F_{n+1}$ $= 5F_{n+1} + 3F_n$	M1	3.1a	Using (not just stating) Fibonacci r.r. at least twice (up or down)
			M1 A1	1.1 1.1	Use of Fibonacci r.r. all the way to $F_{n+1}$ and $F_n$
			[3]		
(ii)		$F_0 = 0$ which is divisible by 5 Assume that $F_{5k}$ is divisible by 5 Then $F_{5k+5} = 5F_{5k+1} + 3F_{5k}$ which is the sum of two terms divisible by 5 (the second by hypothesis) and hence also divisible by 5 Since the result is true for $n = 0$ (allow $n = 5$ ) & true for $n = 5k \Rightarrow$ true for $n = 5(k + 1)$ it follows that the result is true for all non- negative (allow “positive”) integers $n$	B1 B1 M1	1.1a 2.1 3.1a	Allow $F_5 = 5$ as a baseline case  Use of (i)’s result
			A1	2.5	
			E1	2.4	Only given if all 4 other marks earned
			[5]		
		<b>Alt.</b> Consider $\{F_n\}$ modulo 5: $F_0 = 0, F_1 = 1 \Rightarrow F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 0,$ $F_6 = 3, F_7 = 3, F_8 = 1, F_9 = 4, F_{10} = 0,$ $F_{11} = 4, F_{12} = 4, F_{13} = 3, F_{14} = 2, F_{15} = 0,$ $F_{16} = 2, F_{17} = 2, F_{18} = 4, F_{19} = 1, F_{20} = 0, F_{21} = 1$ Since $\{F_n\}$ is given by a second-order, linear, homogeneous r.r. and $F_0 = F_{20} = 0$ , and $F_1 = F_{21} = 1$ , it follows that the sequence mod 5 is periodic with period 20 As $F_0, F_5, F_{10}$ and $F_{15}$ are all $\equiv 0 \pmod{5}$ and the sequence repeats every 20 terms, it follows that $F_{5n} \equiv 0 \pmod{5}$ for all integers $n = 0, 1, 2, \dots$			Note that there are only ${}^5P_2 = 5 \times 4 = 20$ pairs of the five possible residues mod 5; thus the <i>maximum</i> period of the sequence $\{F_n \pmod{5}\}$ is 20.

Y535

Mark Scheme

June 2018

Question		Answer	Marks	AO	Guidance	
7	(i)	(a)	A circle	<b>B1</b>	<b>1.1</b>	If axes shown, must be $x, y$
				[1]		
		(b)	$z = c$ <b>or</b> $f(x, y) = c$	<b>B1</b>	<b>3.3</b>	Not necessary to say that $c$ (or $d$ ) $> 0$ Allow specific positive $c$ values
				[1]		
	(ii)	(a)	A parabola through $O$	<b>B1</b>	<b>1.1</b>	The parabola should stop at $P$ but condone continuations
				[1]		
		(b)	$z = kx^2$ <b>or</b> $z = f(x, 0)$	<b>B1</b>	<b>1.1</b>	Allow specific positive $k$ values
				[1]		
	(iii)		$a$ and $b$ must be positive (since $z \geq 0$ ) $a = b$ by symmetry	<b>B1</b> <b>B1</b> <b>E1</b>	<b>3.3</b> <b>3.4</b> <b>2.4</b>	Statement Justification
				[3]		
(iv)		In (i) (b), $c = 0.065$ $2\pi x = 2.652 \Rightarrow x$ -coordinate of (e.g.) $P$ is 0.422 Thus $a = b = \frac{0.065}{x^2} = 0.365$ (to 3s.f.)	<b>B1</b> <b>B1</b> <b>M1</b> <b>A1</b>	<b>1.1</b> <b>1.1</b> <b>3.1a</b> <b>1.1</b>	Or diameter is 0.844 Using this to find $a, b$ <b>FT</b> their $x$ -coordinate of $P$ Allow unsaid that $k = a$ in (ii) (b)	
			[4]		Or $d = 0.422^2 = 0.178$ <b>FT</b>  No need to note $b = a$ if already noted in (iii)	
(v)		Draw at least two beams which after being reflected meet in $B$ Either the position of $B$ moves accordingly <b>or</b> the beams will miss the receiver <b>o.e.</b>	<b>B1</b> <b>B1</b>	<b>3.2a</b> <b>3.5a</b>	It is not sufficient to say only that the parabola changes shape	
			[2]			

**OCR (Oxford Cambridge and RSA Examinations)**  
**The Triangle Building**  
**Shaftesbury Road**  
**Cambridge**  
**CB2 8EA**

**OCR Customer Contact Centre**

**Education and Learning**

Telephone: 01223 553998

Facsimile: 01223 552627

Email: [general.qualifications@ocr.org.uk](mailto:general.qualifications@ocr.org.uk)

[www.ocr.org.uk](http://www.ocr.org.uk)

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

**Oxford Cambridge and RSA Examinations**  
is a Company Limited by Guarantee  
Registered in England  
Registered Office; The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA  
Registered Company Number: 3484466  
OCR is an exempt Charity

**OCR (Oxford Cambridge and RSA Examinations)**  
Head office  
Telephone: 01223 552552  
Facsimile: 01223 552553

© OCR 2018

 **Cambridge  
Assessment**

